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STUDY OF THE DYNAMICS OF A VIBRATION MACHINE CONSIDERING THE INFLUENCE OF THE PROCESSING MEDIUM

ABSTRACT. *This paper presents the results of studying the dynamics of a vibration machine, taking into account its interaction with the processing medium based on representing the medium as a continuous system. The application of the complex number method significantly simplifies the formation of equations of motion for the discrete-continuous system, which is a computational model of the vibration system "machine-medium." The study substantiates and develops a method for considering the mutual influence of the machine and the processing medium, based on representing the medium as a continuous system. The dynamics of the vibration system "machine-medium" is studied, and oscillation parameters are determined without considering resistance forces. It is found that the overall motion of the vibration system is described by four components. The first three describe the natural oscillations of the system, of which the first two are determined only by initial conditions, and the third reflects the accompanying oscillations caused by the external force applied to the system. The last component defines the forced oscillations following the external force's change law. This result shows that the oscillations of the vibration system are not strictly harmonic, which is confirmed by the provided graphs. The dynamics of the "machine-medium" vibration system, considering resistance forces, are studied, and analytical dependencies for determining oscillation amplitudes and natural*

and resonant frequencies are obtained. The results of the amplitude calculations of the vibration platform for various heights of compacted concrete mixtures reveal the influence of the resistance coefficient and the ratio of oscillation frequency to the wave propagation speed in the mixture. Analysis of the obtained graphs shows that the resistance coefficient has a different effect on the amplitude of oscillations for different heights of the compacting mixture. Certain mixture height zones of the vibration system "machine-medium" operate in a near-resonant mode. A significant influence on the amplitude is exerted by the wave propagation speed, which is included in the analytical formulas for determining the numerical values of the coefficients accounting for reactive and active components of resistance.

Keywords: complex number method, vibration machine, discrete system, concrete mixture, continuous system, joint motion, resistance forces, oscillation parameters, amplitude, frequency.

1. Introduction. Modern demands in the construction industry require minimizing energy consumption while achieving high-quality execution of technological processes during the formation of concrete and reinforced concrete products. The dominant process in the production of these products is the compaction of concrete mixtures using vibration technology. Ensuring effective operating modes and parameters requires using computational models that adequately reflect the real process of vibration compaction. However, in practice, the full achievement of such conditions is constrained by discrepancies between calculated and actual parameters. This is due to the complexity of the processes occurring in the compacted mixture and the use of empirical formulas that are reliable only within the framework of the assumptions and conditions under which these studies were conducted. Moreover, resonant operating modes, which are the most common, are also energy-intensive. Therefore, finding more effective research methods and developing algorithms and calculation methods is a relevant task.

2. Literature Review and Problem Formulation. The complexity of the processes occurring in the compacted mixture under the action of vibration is the reason for various modeling methods, assumptions, and parameter evaluations of efficiency. In [1], when calculating a vibration machine for manufacturing flat concrete slabs, a discrete model was used for both the machine and the compacted mixture, providing a simplified methodology for calculating parameters. The process of compacting cement concrete mixtures in a harmonic mode is presented in [2]. Studies on the dynamics of vibration technology in processes of crushing, sorting, and compacting using both discrete and continuous models are presented in [3,4]. The use of vibration technology for concrete compaction is discussed in [5]. An analysis of the cited works demonstrates the dominance of studies focused on the steady-state operation of vibration technology. The challenge remains to explore more efficient research methods that take into account the influence of the processing material, which is the primary objective of this study.

3. Research Objectives. The goal of this research is to determine the parameters of a vibration machine considering the influence of the processing medium, represented as a continuous system. To achieve this goal, the following tasks were set:

- develop a discrete-continuous model for the motion of the combined dynamic system "vibration machine – compacting medium";
- study the dynamics of the "machine-medium" vibration system and determine the parameters without considering resistance forces;
- study the dynamics of the "machine-medium" vibration system and determine the parameters considering resistance forces.

4. Research and Determination of the Parameters of the Vibration Machine Considering the Influence of the Processing Medium Represented as a Continuous System.

4.1. Development of a Discrete-Continuous Model for the Motion of the Combined Dynamic System "Vibration Machine – Compacting Medium". To compose the motion equation of the machine, considering the influence of the processing medium, the method of complex numbers is applied [3]. The computational diagram of this system is shown in Figure 1.

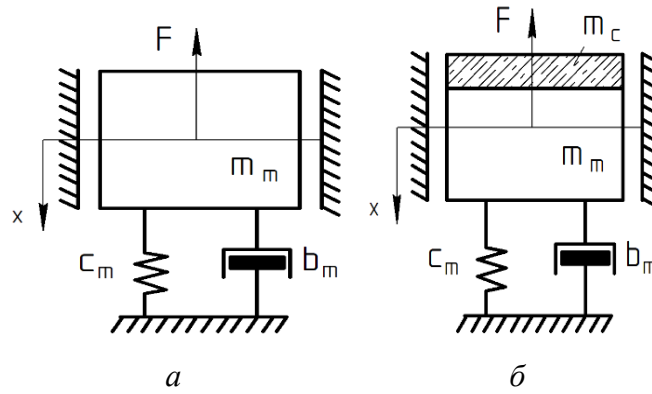


Fig. 1. Computational diagram: a - vibration machine;
b - vibration system "machine-medium".

The natural motion of such a vibration machine is described by the following equation of motion [1]:

$$m_m \ddot{x} + b_m \dot{x} + c_m x = F_0 \cos(\omega t + \varphi), \quad (1)$$

where the first term $m_m \ddot{x}$ represents the inertia force of the moving mass, and the other terms: $b_m \dot{x}$ - represent resistance forces, $c_m x$ - stiffness, and the external force. $F = F_0 \cos \omega t$ To apply the method of complex quantities in equation (1), we replace: $F_0 \cos \omega t$ with $F_0 e^{i\omega t}$ and substituting $\bar{X} e^{i\omega t}$ instead of x , we obtain a formula that relates the complex amplitudes of forces:

$$\bar{X}(-m_m \omega^2 + i\omega b_m + c_m) = F_0 \quad (2)$$

The complex quantity in the brackets of equality (2) is the complex, dynamic rigidity of the system. Let's denote it by \bar{K}_m ; Then equality (2) can be written as follows:

$$K_m = K_m^p + iK_m^a = (c_m - m_m \omega^2) + i b_m \omega. \quad (4)$$

Here $K_m^p = (c_m - m_m \omega^2)$ - the reactive component of resistance; $K_m^a = b_m \omega$ - the active component of resistance. Therefore, the magnitude of the complex amplitude of the force \bar{X} is equal to the amplitude of the force F_0 divided by the dynamic stiffness of the system \bar{K}_m :

$$\bar{X} \cdot \bar{K}_m = F_0 \text{ або } \bar{X} = \frac{F_0}{\bar{K}_m}. \quad (3)$$

If in any oscillatory system there is mass, elasticity, and resistance (discrete system), then it is obvious that any layer of the processing medium (continuum system) has the same properties. Then the reactive K_c^p and active K_c^a components of the dynamic rigidity of the medium will be distributed throughout its volume. Thus, the dynamic rigidity of the "machine-environment" system can be represented on the basis of the use of the rule of addition of complex quantities:

$$K = K_m + K_c = (K_m^p + K_c^p) + i(K_m^a + K_c^a). \quad (5)$$

Knowing the system's dynamic stiffness allows determining the system's response $R = Kx^*$. From the condition of dynamic equilibrium, we derive the displacement law of the "machine-medium" system:

$$x^* = \frac{F_0 e^{i\omega t}}{|K|} = \frac{F_0 e^{i\omega t}}{|K_m + K_c|}. \quad (6)$$

Thus, the problem of considering the processing medium reduces to determining the active and reactive forces in the contact zone and incorporating them into the motion equation of the hybrid dynamic system. In [3], analytical dependencies were obtained for the coefficients accounting for wave phenomena in the continuous system in discrete form:

$$a = \frac{\alpha \operatorname{sh} 2\alpha h + \beta \sin 2\beta h}{h(\alpha^2 + \beta^2)[ch 2\alpha h + \cos 2\beta h]}; \quad d = \frac{\alpha \sin 2\beta h - \beta \operatorname{sh} 2\alpha h}{h(\alpha^2 + \beta^2)[ch 2\alpha h + \cos 2\beta h]}, \quad (7)$$

where a - is the coefficient accounting for the influence of reactive forces in the medium, and d is the coefficient accounting for the influence of active forces in the medium.

Thus, the equation of motion for the "machine-medium" system (Figure 1b) will have the following form:

$$(m_m + m_c a)\ddot{x} + (b_m + d/\omega)\dot{x} + c_m x = F_0 \cos(\omega t + \varphi) \quad (8)$$

The solution of equation (8) in the form: $x = x_0 \cos(\omega t + \varphi)$ we obtain the expression for the amplitude of oscillations, x_0 , of the "machine-environment" system:

$$x_0 = \frac{F_0}{\sqrt{(c_m - m_m \omega^2)^2 + (m_c \omega^2 a)^2 + (b_m \omega)^2 + (m_c \omega^2 d)^2}}, \quad (9)$$

where the denominator (9) represents the dynamic stiffness of the "machine-medium" system in its explicit form:

$$K = \sqrt{(c_m - m_m \omega^2)^2 + (m_c \omega^2 a)^2 + (b_m \omega)^2 + (m_c \omega^2 d)^2} \quad (10)$$

4.2. Study of the Dynamics of the "Machine-Medium" Vibration System and Determination of Parameters Without Considering Resistance Forces. Figure 2 presents the computational diagram of the "machine-medium" vibration system without considering resistance forces.

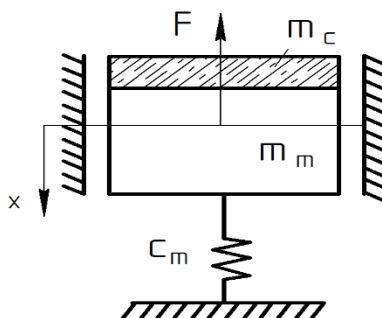


Fig. 2. Computational diagram of the "machine-medium" vibration system without considering resistance forces.

In this case, the equation of motion for such a "machine-medium" system is:

$$m\ddot{x} + c_m x = F_0 \cos \omega t, \quad (11)$$

where $m = m_m + m_c a$.

Equal (11) is converted by dividing each term by the mass:

$$\ddot{x} + \omega_0^2 x = \frac{1}{m} F_0 \cos \omega t, \quad (12)$$

where ω_0 - is the natural frequency of the "machine-medium" system.

$$\omega_0 = \sqrt{\frac{c_m}{m}} \quad (13)$$

The solution to equation (12) is given by [1] and consists of two parts:

$$x(t) = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t + x_0 \cos \omega t. \quad (14)$$

Solution(14) takes into account free oscillations $A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$ and forced oscillations $x_0 \cos \omega t$; A_1 , A_2 are constant integrations, which can be defined by substituting the initial conditions into the integral:

$$[x(t=0) = x_0, \quad \dot{x}(t=0) = \dot{x}]. \quad (15)$$

Substituting the solution (14), which includes forced oscillations, into equation (12), we obtain the formula for the oscillation amplitude in the steady-state mode:

$$x_0 = F_0 / m(\omega_0^2 - \omega^2). \quad (16)$$

Taking into account equation (16), the general solution (14) becomes:

$$x(t) = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t + \frac{F_0 \cos \omega t}{m(\omega_0^2 - \omega^2)} \quad (17)$$

The derivative of equation (17) with respect to time gives the expression for the velocity of the "machine-medium" vibration system:

$$\dot{x} = -A_1 \omega_0 \sin \omega_0 t + A_2 \omega_0 \cos \omega_0 t - 1 \frac{F_0 \omega}{m(\omega_0^2 - \omega^2)} \sin \omega t. \quad (18)$$

Now we can obtain the expressions of the coefficients A_1 , A_2 by substituting the initial conditions (15) in the solution (17) and (18) we have:

$$A_1 = x_0 - \frac{F_0}{m(\omega_0^2 - \omega^2)}; \quad A_2 = \frac{\dot{x}_0}{\omega_0}. \quad (19)$$

Finally, we can write the equation describing the motion of the "machine-medium" vibration system:

$$x(t) = x_0 \cos \omega_0 t + \frac{\dot{x}_0}{\omega_0} \sin \omega_0 t - \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos \omega_0 t + \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos \omega t. \quad (20)$$

The resulting dependence (20) shows that, in general, the motion of a vibrational system is described by three terms. The first three terms describe the proper oscillations of the system, of which the first two are determined only by the initial conditions, and the third reflects the concomitant oscillations by the external force applied to the system. The last term of expression (20) defines forced oscillations according to the law of change of external force. Forced oscillation frequency ω and natural oscillation ω_0 frequency are different from each other, so the oscillations described by expression (20) are not strictly harmonic, which is confirmed by the above graphs (Fig. 3), where x_a and φ_0 is the amplitude and the initial phase of free oscillations

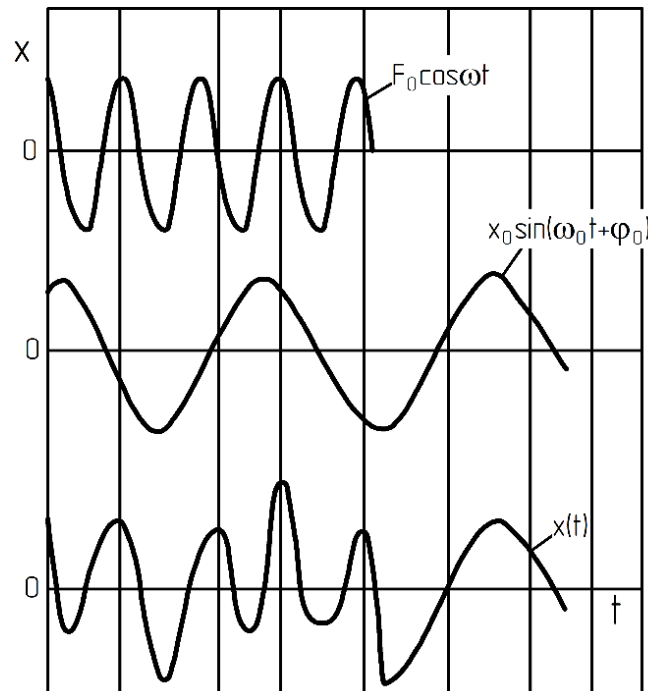


Fig. 3. Graph of the combined oscillations of the "machine-medium" vibration system without considering resistance forces.

Thus, the total oscillation of the system (1.1) consists of the sum of the unquenchable natural oscillations that last indefinitely and the forced oscillations that last during the time of the external force. Simultaneously with the forced oscillations, the accompanying oscillations also cease. The amplitude of oscillations x_0 and the phase angle φ_0 are determined by the following formulas:

$$x_0 = \sqrt{\left[x_0 - \frac{F_0}{m(\omega_0^2 - \omega^2)} \right]^2 + \left(\frac{\dot{x}_0}{\omega_0} \right)^2};$$

$$\varphi_0 = \text{arctg} \frac{\left(x_0 - \frac{F_a}{m(\omega_0^2 - \omega^2)} \right) \omega_0}{\dot{x}_0} \tag{21}$$

The following conclusions can be drawn from the formulas obtained.

1. Provided that the frequency of forced oscillations ω is less than the frequency of natural oscillations ω_0 , $\omega < \omega_0$ the amplitude of forced oscillations x_0 is in the same phase with the external force F_0 , which, with the component of the inertial forces of the system, is balanced by the elastic forces. That is, the system is in the pre-resonance zone.

2. Provided that the frequency of forced oscillations ω and intrinsic ω_0 oscillations is equal.: $\omega = \omega_0$ Resonance occurs in the system, which makes it possible to obtain oscillations of the system with large amplitudes at minimum values of the amplitude of the external force.

3. Provided that the frequency of forced oscillations ω is greater than the frequency of natural oscillations ω_0 , $\omega > \omega_0$ the amplitude of forced oscillations x_0 is in the opposite phase with the external force F_0 , which is balanced with the elastic forces by the inertial forces of the system. That is, the system is located in the resonance zone. Forced oscillations coincide in phase with the forcing force, and when $\omega > \omega_0$ are in the opposite phase.

4.3. Study of the Dynamics of the "Machine-Medium" Vibration System and Determination of Parameters Considering Resistance Forces. The motion of the vibration system "machine – medium" taking into account the resistance forces (Fig. 1, a) is described by the above equation (1), which, after dividing by the mass of the system, will look like this:

$$\ddot{x} + 2h\dot{x} + \omega_0^2 x = \frac{F_a}{m} \cos(\omega t + \varphi), \quad (22)$$

where h is the energy dissipation coefficient: $h = b/2m$

The solution of equation (22) is as follows

$$x(t) = (A_1 \cos \omega_0 t + A_2 \sin \omega_0 t) e^{-ht} + x_0 \cos(\omega t - \varphi), \quad (23)$$

Where the coefficients A_1, A_2 are already defined above(19); x_0 , is still the amplitude of forced oscillations; φ - the phase between the external force and the amplitude of forced oscillations of the vibration system.

By substituting solution (23) into equation (22) after simple transformations, we obtain formulas for determining the amplitude of oscillations x_a and phase

$$\left. \begin{aligned} x_a &= \frac{F_a}{m \sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2 \omega^2}}; \\ \varphi &= \operatorname{arctg} \frac{2h\omega}{\omega_0^2 - \omega^2}. \end{aligned} \right\} \quad (24)$$

Substituting (24) A_1, A_2, ω_0, x_0 i φ into equation (23) we obtain an expression describing the general motion of the system:

$$\begin{aligned} x(t) &= \left(x_0 \cos \omega_0 t + \frac{x_0 h + \dot{x}_0}{\omega_1} \sin \omega_0 t \right) e^{-ht} - \\ &- \frac{F_a e^{-ht}}{m \left[(\omega_0^2 - \omega^2)^2 + 4h^2 \omega^2 \right]} \left[(\omega_0^2 - \omega^2) \cos \omega_0 t + \right. \\ &+ \left. \frac{h}{\omega_1} (\omega_0^2 + \omega^2) \sin \omega_0 t \right] + \frac{F_a \cos(\omega t - \varphi)}{m \sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2 \omega^2}}. \end{aligned} \quad (25)$$

In equation (25), the structure of the general solution is similar to that in equation (20): the first term represents the initial free oscillations, determined only by the initial conditions; the second term represents the accompanying natural oscillations determined by the external force, and the third term represents the forced oscillations. The difference between solution (25) and solution (20) lies in the presence of terms accounting for energy dissipation in equation (25). In such a system, the natural oscillations decay quickly, and only the forced oscillations remain in the system (Figure 4).

Over time, only forced oscillations remain in the system, described by the following expression:

$$x(t) = \frac{F_a \cos(\omega t - \varphi)}{m \sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2 \omega^2}} \quad (26)$$

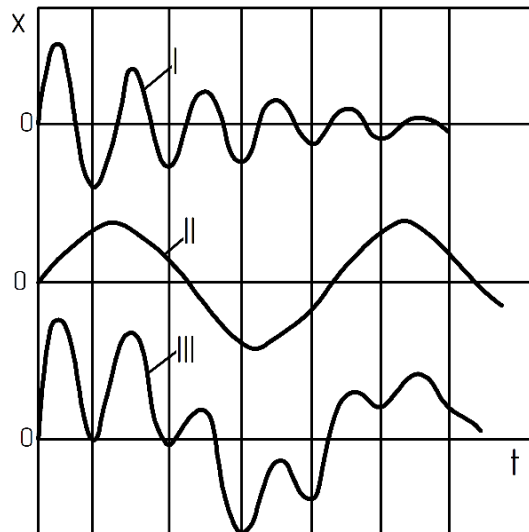


Fig. 4. Graphs of oscillations in the "machine-medium" vibration system considering resistance forces: I - decaying oscillations; II - forced oscillations; III - combined oscillations.

The resonant frequency, considering resistance forces, is determined by the formula:

$$\omega_p = \sqrt{\omega_0^2 - 2h^2}. \tag{27}$$

From this formula it follows that the resonant frequency ω_p is less than the natural frequency ω_0 : $\omega_p < \omega_0$, which is evidenced by the above graph (Fig. 5)

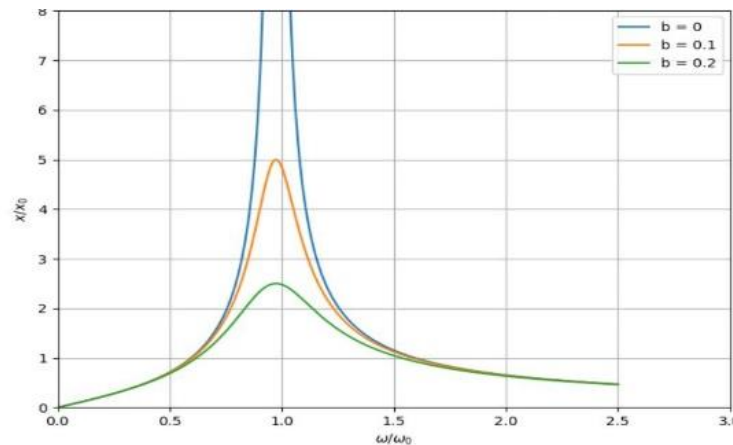


Fig. 5. Graph of changes in oscillation amplitude depending on frequency for different values of the energy dissipation coefficient.

Now, let us determine the degree of influence of coefficients (7) representing the reactive and active components of resistance in the compacting medium on the amplitude of oscillations in the vibration machine (9). Figure 6 presents the results of calculations regarding the influence of the resistance coefficient γ and the ratio of oscillation frequency to wave propagation speed c on the amplitude of oscillations in the vibration platform for different heights of the compacted concrete mixture.

From the graphs (Fig. 6, a) it follows that the coefficient γ has an insignificant effect on the magnitude of the amplitude of oscillations x at the height $h = 0.10$ m $h = 0.20$ γ . For height m , the $h = 0.3$ amplitude of oscillations decreases with increasing γ , which is explained by the significant influence of dissipative resistance forces, since in such a situation the vibration system "machine – medium" operates in an approximate resonance mode. A noticeable influence on the amplitude is

exerted by the speed of propagation of oscillations (Fig. 6.b), the value of which is included in the determination of the numerical values of the resistance coefficients of dependence (7).

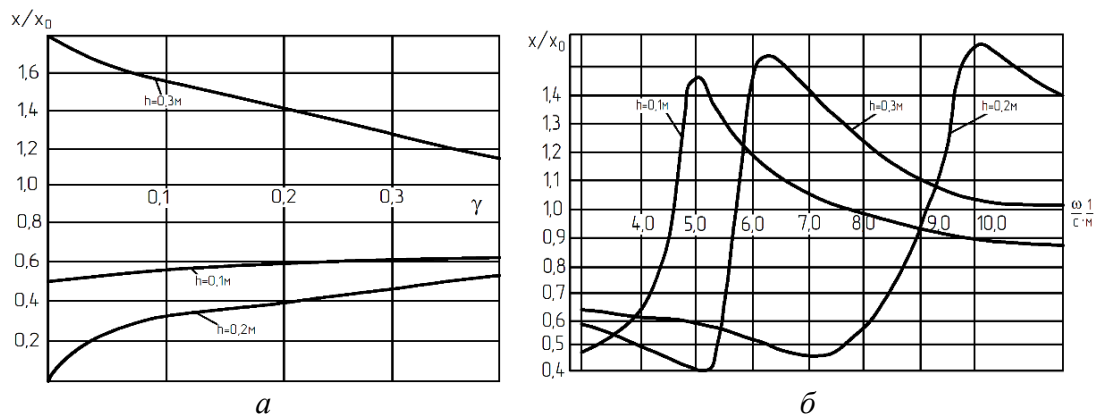


Fig.6. Results of calculations of the amplitude of vibrations of the vibration platform at different heights of the compaction concrete mixture: a-effect of the resistance coefficient γ ; The ratio of the frequency of

oscillations ω to the velocity of propagation of waves C in a mixture $\frac{\omega}{c}$.

5. Discussion of the results. It was found that, in general, the motion of the vibrational system (Fig. 1) is described by equation (20) with four terms. The first three describe the natural oscillations of the system, of which the first two are determined only by the initial conditions, and the third reflects the concomitant oscillations by the external force applied to the system. The last term defines forced oscillations according to the law of change of external force. This result is new and shows that the vibrations of the vibration system are not strictly harmonic, which is confirmed by the above graphs. The dynamics of movement of the vibration system "machine – environment" is investigated, taking into account the resistance forces. Analytical dependencies for determining the amplitude of oscillations and natural and resonant frequencies of oscillations have been obtained. The degree of influence of the coefficients (7) of the reactive and active components of the resistance of the sealing medium on the amplitude of oscillations of the vibration machine (9) is determined. From the graphs (Fig. 6, a) it follows that the coefficient γ has an insignificant effect on the magnitude of the amplitude of oscillations x at the height $h = 0,10$ m $h = 0,20$ γ . For height m, the $h = 0,3$ amplitude of oscillations decreases with increasing γ , which is explained by the significant influence of dissipative resistance forces, since in such a situation the vibration system "machine – medium" operates in an approximate resonance mode. A noticeable influence on the amplitude is exerted by the speed of propagation of oscillations (Fig. 6.b), the value of which is included in the determination of the numerical values of the resistance coefficients a, d of dependence (7).

6. Conclusions.

1. It was established that ensuring effective operating modes and parameters is constrained by discrepancies between calculated and actual parameters. This is due to the complexity of the processes occurring in the compacted mixture and the use of empirical formulas that are valid only within the assumptions and conditions under which the research was conducted.

2. A research method was proposed that considers the vibration system "machine-medium" as a system governed by a single vibration process. The development of the model for this combined dynamic system is based on modern advances in the classical theory of mechanical oscillations for the subsystem "vibration machine," and the compacting medium subsystem is modeled based on the theory of dispersed media as continuous models.

3. Analytical dependencies were obtained for determining the oscillation amplitude and the natural and resonant frequencies of oscillations. The degree of influence of the reactive and active components of resistance in the compacted medium on the oscillation amplitude of the vibration machine was determined.

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